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THEORETICAL DESIGN OF A VARIABLE ACTIVATION ENERGY TIME-TEMPERATURE INTEGRATOR FOR PREDICTION OF FOOD OR DRUG SHELF LIFE

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ABSTRACT

Successful application of a time/temperature integrator (TTI) requires that it has an activation energy similar to the major deterioration reaction of the food or drug it is to monitor. Based on the 3M Monitor Mark, a new tag was designed, the principle of which is a simultaneous diffusion and chemical reaction process. The combined process could be modelled by a response function $f(x) = x^2 = k_c t^*$. The overall activation energy could be increased (compared to the simple diffusion process) if the activation energy and half life for the chemical reaction are chosen appropriately. An equation was developed to estimate the amount of diffusing substance needed. Theoretically, it is feasible to design a new tag that will have essentially the same activation energy as food or drug quality loss.

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INTRODUCTION

The proper use of a time-temperature indicator (TTI) could facilitate scheduling of distribution such that products approaching the end of their shelf life are moved first. This would reduce food waste, ensure that a higher quality food product reaches the consumer and that a drug has not lost its potency and indicate problem areas in the distribution system so that they could be resolved [1]. For example, the World Health Organization began requiring TTI use on each case lot after finding that much of the vaccine distributed for use in lesser developed countries had been exposed to abusive temperature conditions such that they lost their potency.

There are several reasons for limited use of TTI for foods especially including the high cost, reliability of the tag and questions of applicability. Taoukis and Labuza [2,3] addressed these issues for several TTIs. They found that the reliability of a TTI depends on the correlation of the TTI temperature response with that of reactions leading to quality loss. Unless the change in the rate with temperature of the TTI device closely parallels the temperature dependence of the rate of quality deterioration of the monitored product, the device will not be able to accurately predict the shelflife remaining for a variable temperature distribution. Of critical importance is the fact that the outcome of the tag response is an effective temperature for the time period at which it is measured. This effective temperature (T_{eff}) is the constant temperature, which if the tag were always at it, that would give the same measured response as for the variable temperature exposure. As shown by Taoukis and Labuza [3], this T_{eff} is a function of the temperature dependent model (temperature sensitivity) of the tag. It is used in the application scheme by applying it to the reaction model for the reaction causing deterioration, using the same temperature sensitivity model to calculate the amount of shelf life remaining. Thus the T_{eff} is an integrated temperature function for the actual distribution.

The temperature sensitivity of chemical reactions causing deterioration in foods or drugs can be evaluated using the Arrhenius relationship where the natural log of the rate

TABLE 1
Typical E_A Values for Quality Losses[a]

Reactions	Activation energy (kcal/mol)
Acid/base catalysis	20-30
Enzymic reactions	10-15
Hydrolytic reactions	15-20
Lipid oxidation	10-25
Nutrient losses (e.g. Vitamin C)	20-30
Maillard reaction	25-50
Microbial growth	20-60

a. rate constant $k = k_o \exp[-E_A/(RT)]$ where R = gas constant.

constant , k, plotted vs. 1/T (K) gives a straight line of slope E_A/R where E_A is the activation energy (kcal/mol) (a measure of the temperature sensitivity). The closer the E_A of the tag is to that of the product being monitored, the less error in the prediction of remaining shelf life [3]. Table 1 lists E_A values for a number of reactions, showing the large range found. Since tags are limited in their chemistry and engineering design, a second problem is that their response at a given temperature may be faster or slower than the rate of the food deterioration reaction. In essence the ideal case would be for the tag kinetics to be the same as that of the food deterioration reaction kinetics. The end point of the tag is that time when either visual or instrumental methods show a particular change, for example the color of the changing point becomes the same as its reference or the color movement along a track reaches the end of the track. In the second case the track, of course, could be made longer if practical.

Figure 1 shows all the possible variations in both response time and temperature sensitivity E_A. Four assumptions have to be made in plotting Fig.1: (1) within the temperature range, both the TTI and the monitored product follow the same reaction rate kinetics generally either pseudo-zero order or pseudo-first order kinetics; (2) the

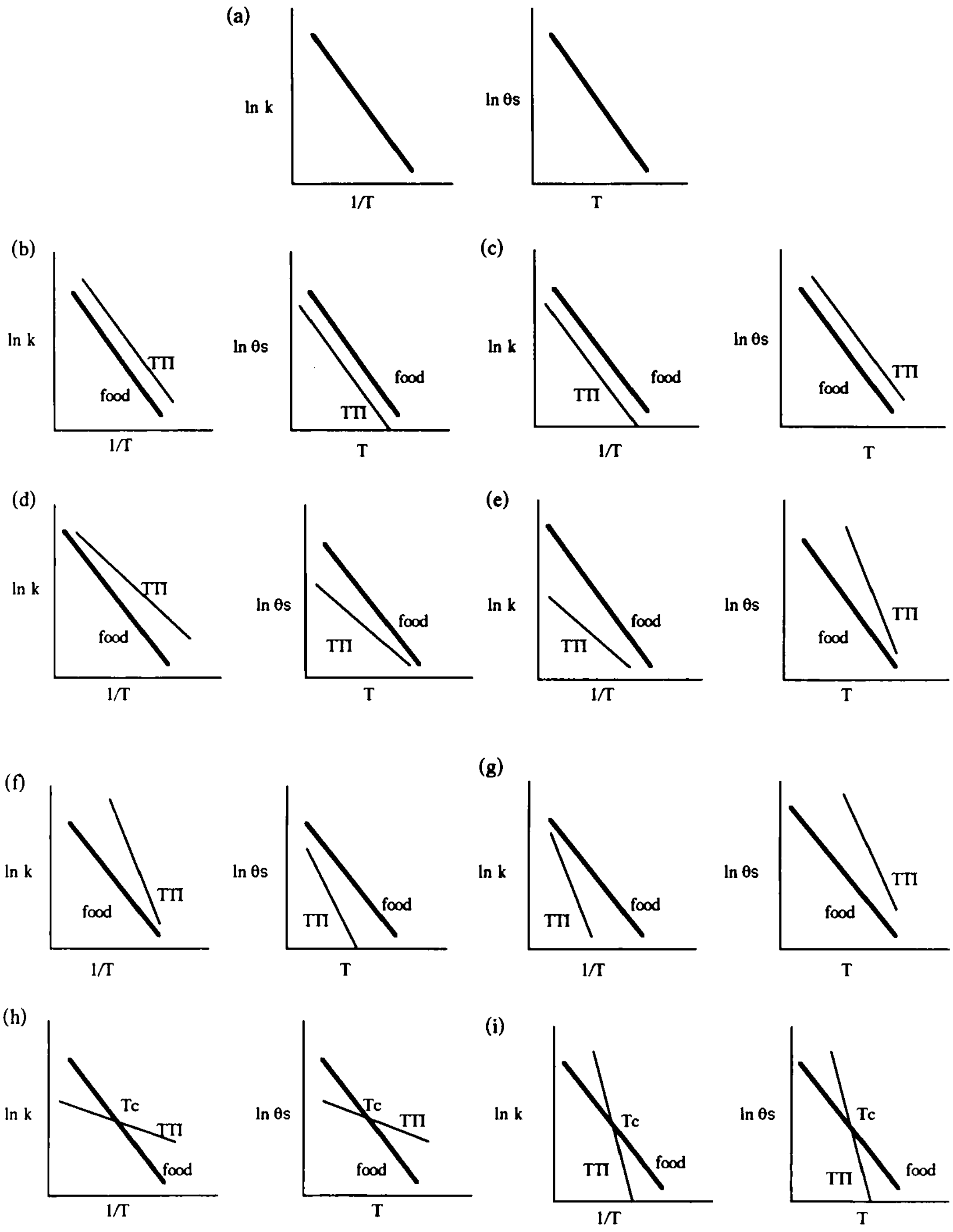

FIGURE 1
Possible Correlations of TTI Response with Food Quality Change Reactions in Foods or Drugs. (Table 2 Describes the Parameters for Each Subfigure.)

temperature sensitivity of both follows the Arrhenius equation; (3) k values for both TTI and food have the same units (e.g. % of change per time or reciprocal time) and (4) the TTI and the reaction show no history effect under a variable time-temperature history [3]. Table 2 lists the selected parameters for each subfigure in Fig. 1, where θ_s is the endpoint time for either the tag or food deterioration reaction, e.g. the time for vitamin C loss to reach the value on a food label or the lower 95% confidence limit of the time to reach the legal label limit (= 90% of original value for a life saving drug).

Basically the optimal would be the case in Fig. (1a) where both times overlap, a condition which is generally not encountered. Condition (1b) is where both have the same E_A and the tag reaches its endpoint before the product does, at all temperatures, therefore indicating an unacceptable product before its true end point, but if the endpoint were based on safety, this would be a good control point. One would hope, of course, that the use or consumption time is less than this end point. In Fig. (1c), the TTI reacts slower at all temperatures, thus unless the food or drug was always consumed before end of shelf life, the tag, would always reach the endpoint after the product becomes illegal or spoiled, unless it gave a easily discernable continuous indication (such as a moving front). Since in both these cases the ratio of the tag rate to the product reaction rate is constant at any temperature, accurate prediction of shelf life consumed at any point in time (for a continuous tag) would still be possible. In (1d) the tag has a faster response and a smaller E_A, resulting in an under prediction of shelf life below some upper temperature limit of applicability, where the tag and reaction response rate are the same. However if shelf life was based on food safety concerns such as the growth of *Clostridium botulinum* in a controlled atmosphere packaged fish product and the food was exposed above that temperature, the food could be unsafe without the tag reaching its endpoint. Under the conditions of Fig. (1e) the product would become unacceptable before the tag endpoint is reached at all temperatures below an upper limit even though the product reaction E_A is larger. The conditions for Figs. (1f) and (1g) indicate a lower temperature limit where the

TABLE 2
Descriptive Explanations to Figure 1

Subfigure	Activation energy relationship	Shelf life relationship	Prediction of shelf life	Comments
(a)	$E_{A(food)} = E_{A(TTI)}$	$\theta_{s(food)} = \theta_{s(TTI)}$	Ideal	unrealistic
(b)	$E_{A(food)} = E_{A(TTI)}$	$\theta_{s(food)} = c\theta_{s(TTI)}$ where $c > 1$	Ideal	tag responses such that wastes good food
(c)	$E_{A(food)} = E_{A(TTI)}$	$\theta_{s(food)} = c\theta_{s(TTI)}$ where $c < 1$	possibly accurate	food spoils before tag end point unless consumed first
(d)	$E_{A(food)} > E_{A(TTI)}$	$\theta_{s(food)} > \theta_{s(TTI)}$	predicts shorter shelf life left	tag reaches end point first below an upper critical temperature
(e)	$E_{A(food)} > E_{A(TTI)}$	$\theta_{s(food)} < \theta_{s(TTI)}$	predicts longer shelf life left	unsafe as tag response longer than food
(f)	$E_{A(food)} < E_{A(TTI)}$	$\theta_{s(food)} > \theta_{s(TTI)}$	predicts shorter shelf life left	same as (d) - always safe above a critical temperature
(g)	$E_{A(food)} > E_{A(TTI)}$	$\theta_{s(food)} < \theta_{s(TTI)}$	predicts longer shelf life left	unsafe as in (e)
(h)	$E_{A(food)} > E_{A(TTI)}$	$\theta_{s(food)} > \theta_{s(TTI)}$ if $T<T_c$ $\theta_{s(food)} < \theta_{s(TTI)}$ if $T>T_c$	complicated complicated	dangerous dangerous
(i)	$E_{A(food)} < E_{A(TTI)}$	$\theta_{s(food)} < \theta_{s(TTI)}$ if $T<T_c$ $\theta_{s(food)} > \theta_{s(TTI)}$ if $T>T_c$	complicated complicated	dangerous dangerous

tag and quality loss response are the same. In (1f) the tag has a higher E_A and thus would predict a shorter shelf life above the critical temperature, always leading to a safe condition, but with the potential for always discarding good product if it is not consumed by this time. The opposite is true for condition (1g). In Figs. (1h) and (1i) the conditions indicate that the tag and the food have the same response at a critical temperature that is somewhere within the middle of the storage temperature range rather than at either end. These are the worst possible conditions, since the lines cross. Since the tag is used to integrate the actual temperature distribution, application of the T_{eff} to the food would be complicated and could result in very dangerous predictions for drug or food safety situations.

Rodriguez and Zaritzky [4] developed a time-temperature integrator-indicator for frozen beef, based on the diffusion of a dye in gel. They considered, as an essential condition, that the activation energy for the device shall coincide with the activation energy of frozen beef deterioration. They reported a quite comparable result with $E_{A(TTI)} = 9.2$ kcal/mol and $E_{A(beef)} = 9.4$ kcal/mol. For most foods and drugs where TTIs would be applicable, the activation energy values for shelf life limitation varies between 15-35 kcal/mol [5,6]. For practical use, if the difference in activation energy between the reaction leading to quality loss and the TTI is within ± 2 kcal/mol, the error caused by the lack of correlation is small. Taoukis and Labuza [2] reported that $|E_{A(food)} - E_{A(TTI)}| < 10$ kcal/mol will result in an error of the quality estimation of less than 15%, which in some cases may be acceptable [7].

The performance of three indicator types: 3M, I-Point and Lifelines have been studied by several workers [2,3,8]. The I-Point® TTI is based on a color change caused by a pH decrease, due to a controlled enzymatic hydrolysis of a lipid substrate. It was described in detail by Blixt and Tiru [9], and models with activation energies of around 15, 20 and 35 kcal/mol were available at one time. The Lifelines Freshness Monitor® is based on the solid state polymerization of a thinly coated colorless acetylenic monomer that changes to a highly colored polymer. It has been described in more detail by Fields and Prusik [10], and models with E_As of 20 and 25 kcal/mol are available. The 3M Monitor Mark® system is based on a time-temperature dependent diffusion of a dye through a porous wick of a fatty acid ester. The measurable color advance along the wick is related to the diffusion coefficient of the dye in the supporting medium such that the quantified advance over a graduated scale will be proportional to the loss in quality of the food concerned. The length of the wick determines the shelf life of the tag itself. It has been described in detail by Manske [11] and was patented by the 3M company (St. Paul, MN). Its first and most significant application was to monitor refrigerated vaccine shipments by the World Health Organization (WHO). However, the main drawback is the lower activation energy of around 10-12 kcal/mol which is the result of a diffusion process [2].

The purpose of this study was to investigate whether it is theoretically possible to modify the diffusion based tag to create approximately the same activation energy as that of the quality loss reactions in foods or drugs through a simultaneous chemical reaction occurring on the wick. If possible, the new TTI could have wider applicability to different degradation mechanisms by choosing different reacting substrates, if the chemistry were known.

METHODOLOGY

Defining the Diffusion System in the Current 3M Monitor Mark®

The system consists of a reservoir of a wax containing a visible dye impregnated on the end of a rectangular absorbent wick. Above a critical temperature (the melting point, which can be set by choice), the wax and dye diffuse down the wick. The diffusion along the wick can be mathematically approximated by equations for diffusion in a semi-infinite medium. Based on the proper boundary and initial conditions and mathematical manipulations, the diffusion system has been simply described by $f(x) = x^2 = k_d t^*$, where f(x) is a response function of the TTI, x is the distance of diffusion, k_d is the rate constant at a given temperature T for the single diffusion process and t* is the time to reach C*, which is the concentration at the just visible moving front. From experimental data, k_d was determined as the slope of the plot of x^2 (observable distance diffused squared) vs. time for different constant temperatures in the range of 4 - 30°C. One model designed for refrigerated CAP/MAP foods had an E_A of 9.8 kcal/mol [2].

Taoukis and Labuza [2] showed that :

$$k_d = 4\,D\left[\operatorname{erfc}^{-1}\left(\frac{C^*}{C_o}\right)\right]^2 \qquad (1)$$

where D is the diffusion coefficient in mm^2/hr and erfc^{-1} represents an inverse complementary error function.

The New System: Simultaneous Diffusion and Reaction System

In the proposed case of a simultaneous diffusion and reaction system, compound A (which is colorless) diffuses along the wick with a temperature dependent diffusion coefficient related to k_d. A simultaneously reacts with compound B (colorless) that is bound to the track of the wick to produce compound P, which shows a visible color when it reaches concentration P* at the visually observable moving front. This is depicted in Fig. 2.

To characterize the combined process, several assumptions must be made: (1) The initial concentration of compound A (A_o) is kept in excess at the origin; (2) compound B is spread uniformly along the wick and is bound to it so that no backward or forward diffusion of excess B will occur after the reaction of A with B; (3) B is in excess (negligible change of B with time); (4) the product P of the reaction will not diffuse along the wick, i.e., it is bound as reaction component of B and (5) the minimum visible concentration of P is P* which forms the forward moving front.

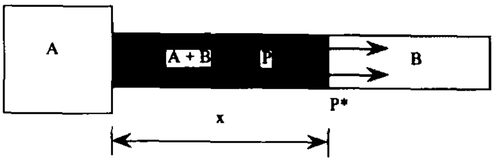

FIGURE 2
Proposed Design for a Diffusion plus Reaction Tag. (A is the Diffusing Substrate Which Reacts with Compound B Bound to the Whole Track Producing a Moving Boundary Front at Conc. P*. B is Bound and does not Diffuse.)

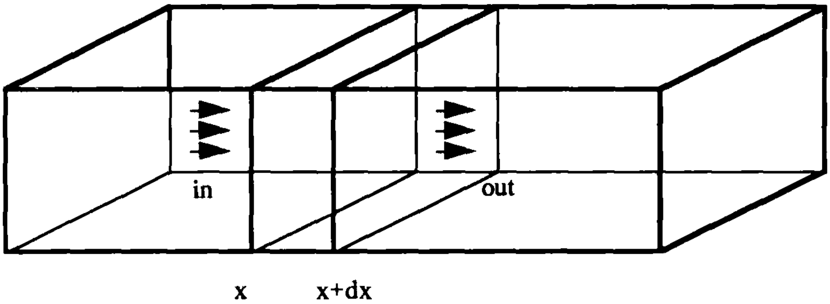

FIGURE 3
An Element of Volume of Unit Cross-Sectional Area between Planes x and x + dx.

If one considers an element of volume of unit cross-sectional area between planes x and x + dx as in Fig. 3 and the initial boundary conditions, an equation for simultaneous diffusion and reaction could be derived and solved [12]. By applying first order kinetics to the reaction of A with B with a rate constant k, for a given constant temperature, an equation for calculating the visible concentration of product can be derived. A BASIC program was written to solve the complicated equation numerically on an IBM-AT personal computer.

TABLE 3
Determination of diffusion coefficient (D)[a]

T	k_d	D (mm^2/hr) for		
(°C)	(mm^2/hr)	C^*/C_o=0.01	C^*/C_o=0.005	C^*/C_o=0.001
4	3.68	0.278	0.233	0.167
10	5.62	0.424	0.357	0.254
13	6.37	0.481	0.404	0.288
20	10.69	0.807	0.678	0.484
25	12.12	0.915	0.769	0.549
30	17.61	1.329	1.117	0.797

a. based on the simple diffusion process.

Estimation of Parameters

Based on previous studies of similar systems [4], the ratio of visible concentration vs. the concentration at the origin (A^*/A_o) was estimated to be between 0.001 to 0.01. From the work of Taoukis and Labuza [2], the diffusion coefficient D was determined from the rate constant k_d, based on different A^*/A_o ratios of 0.001, 0.005, and 0.01. These are shown in Table 3. D values would be expected to have the same Arrhenius type dependence on temperature [13] as the k_d value as found by Taoukis and Labuza [2].

It was assumed that the the chemical reaction would follow first order kinetics with respect to A (B in excess) and have an activation energy value that could be chosen between 20-40 kcal/mol depending on the chemical nature of A and B. No attempt was made to try to determine what A and B should be; however, if A was lysine and B was an aldehydic group on a polymer surface bound to the wick structure, the product P would be a fluorescing Amadori rearrangement product. This reaction should have an E_A of 30-40 kcal/mol. In addition, a half-life for the reaction of between 50-2000 hr was used in the simulations. The rate constant k for a first order reaction is related to the half life by $k = 0.693/\theta_{1/2}$. The estimated parameters for the pure chemical reaction used in the computer simulation are detailed in Table 4.

TABLE 4
Parameters for the Simulated Pure Chemical Reaction Between A and B[a]

		$k_o(hr^{-1})$		
$\theta_{1/2}(hr)$	$k(hr^{-1})$	$E_A= 20^b$	$E_A = 30$	$E_A = 40$
50	0.0139	$1.13x10^{13}$	$3.24x10^{20}$	$9.25x10^{27}$
100	$6.93x10^{-3}$	$5.65x10^{12}$	$1.61x10^{20}$	$4.61x10^{27}$
200	$3.465x10^{-3}$	$2.83x10^{12}$	$8.07x10^{19}$	$2.31x10^{27}$
500	$1.386x10^{-3}$	$1.13x10^{12}$	$3.22x10^{19}$	$9.23x10^{26}$
1000	$6.93x10^{-4}$	$5.65x10^{11}$	$1.61x10^{19}$	$4.61x10^{26}$
2000	$3.465x10^{-4}$	$2.83x10^{11}$	$8.07x10^{18}$	$2.31x10^{26}$

a. $dA/dt = k'AB = kA$ (B_o in excess)
b. E_A in kcal/mol.

RESULTS AND DISCUSSION

Derivation of the Equation for a Simultaneous Diffusion and Chemical Reaction Process

The rate at which compound A crosses unit area of any plane of constant x in the direction of increasing x is

$$\left(\frac{dQ}{dt}\right)_x = - D\frac{\partial A}{\partial x} \qquad (2)$$

Consider an element of volume of unit cross-sectional area between planes x and x+dx (Fig. 3). The following changes in its content of compound A occur in time dt:

Diffusing in :

$$- D\frac{\partial A}{\partial x}dt \qquad (3)$$

Diffusing out:

$$- \left[- Ddt\left(\frac{\partial A}{\partial x} + \frac{\partial^2 A}{\partial x^2}dx \right) \right] \qquad (4)$$

Reacting:

$$- kAdtdx \tag{5}$$

The rate at which chemical reaction destroys the compound per unit volume is k A.

Therefor the net change of A in the volume element is:

$$\left(D\frac{\partial^2 A}{\partial x^2} - kA \right) dtdx \tag{6}$$

If the resulting change in concentration is dA, the net change above can be equal to dA dx, giving

$$\frac{\partial A}{\partial t} = D\frac{\partial^2 A}{\partial x^2} - kt \tag{7}$$

with the boundary and initial conditions:

$A = A_0,\ x = 0,\ t > 0$

$A = 0,\ x = \infty,\ t > 0$

$A = 0,\ x > 0\ ,\ t = 0.$

The above equation can be solved through a Laplace transformation. The solution is also given by Danckwerts [12] as follows:

$$\frac{A}{A_o} = \frac{1}{2}\exp\left(- x\sqrt{\frac{k}{D}} \right) \operatorname{erfc}\left[\frac{x}{2\sqrt{Dt}} - \sqrt{kt} \right] + \frac{1}{2}\exp\left(x\sqrt{\frac{k}{D}} \right) \operatorname{erfc}\left[\frac{x}{2\sqrt{Dt}} + \sqrt{kt} \right] \tag{8}$$

Based on general chemical kinetics, in the case of a first order reaction, the reaction rate is:

$$\frac{dP}{dt} = k[A] \tag{9}$$

where B is in excess. With respect to position then:

$$P(x,t) = k\int_o^t A(x,t)dt \tag{10}$$

When t -> t*, P -> P* (i.e. time t* is the time at any position when the boundary becomes visible (P = P*)). Thus:

$$P^*(x,t^*) = k\int_0^{t^*} A(x,t)\,dt \qquad (11)$$

Substituting Eq. (8) into Eq. (11) then we have the following equation:

$$P^*\left(x,t^*\right) = \frac{A_o}{2}k\Big[\ \exp\left(-x\sqrt{\frac{k}{D}}\right)\int_o^{t^*}\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}} - \sqrt{kt}\right)dt + \exp\left(x\sqrt{\frac{k}{D}}\right)\int_o^{t^*}\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}} + \sqrt{kt}\right)dt\ \Big] \qquad (12)$$

again where t* is the time to reach P* at a given distance x.

Simulation Results

Instead of calculating P* itself in Eq. 12, we are interested in the relationship of the distance of the visible color front (x) along the wick vs. t* plot for a particular diffusion and reaction system. The combined diffusion plus reaction rate constant (k_c) for each temperature was obtained from a linear regression of x^2 vs. t*, and the Arrhenius equation was applied to these slopes. By testing various appropriate parameters in the ranges shown in Tables 3 and 4, a good linear relationship between x^2 and t* was found in the temperature range of 0-30°C. Figure 4 is a plot of x^2 vs. t* for a reaction with a half life $\theta_{1/2(rea)}$ = 500 hr, $P^*/A_o = 0.005$ and $E_{A(rea)}$ = 40 kcal/mol. The combined activation energy for the simultaneous diffusion and reaction system in this case was $E_{A(overall)}$ = 19 kcal/mol compared to the E_A of 9.8 kcal/mol for diffusion alone. The plot is shown in Fig. 5.

The results for a series of simulation conditions are shown in Table 5 and Table 6 for an $E_{A(rea)}$ of 30 and 40 kcal/mol, respectively. It can be seen that the $E_{A(overall)}$

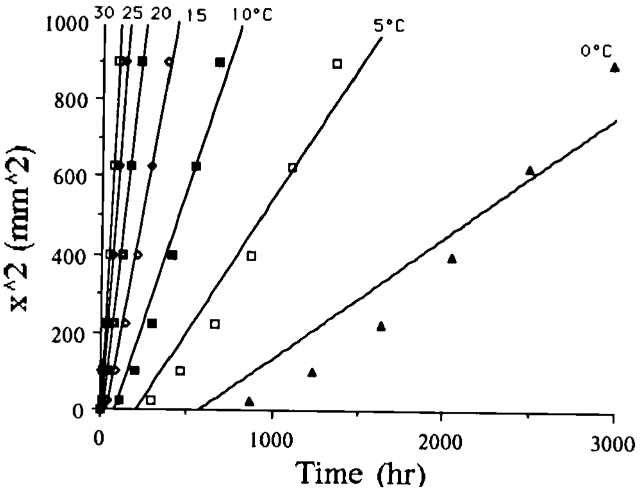

FIGURE 4

A Typical Graph of Distance^2 vs. Time (Activation Energy for Reaction = 40 kcal/mol, Half Life = 500 hr and P^*/A_0 = 0.005).

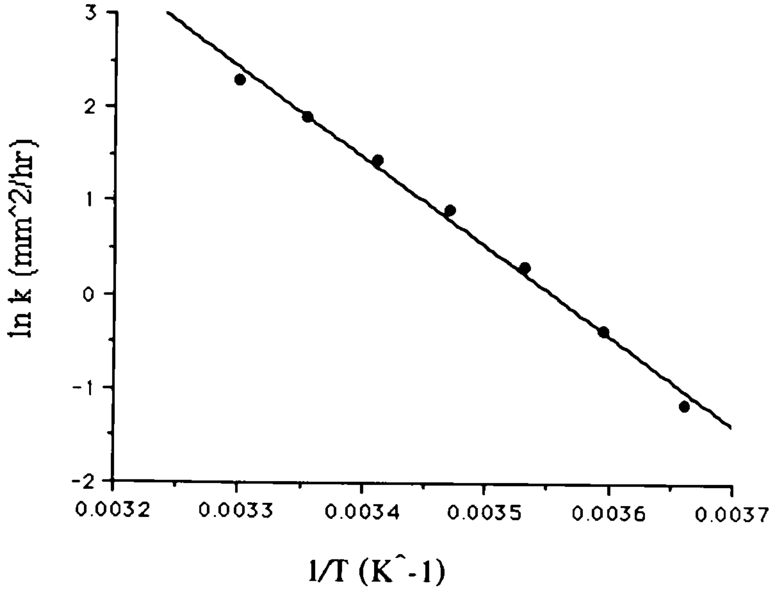

FIGURE 5

A Typical Arrhenius Plot for a Simultaneous Diffusion and Reaction Process Using the Simulation Results from Fig. 4.

TABLE 5
The Overall Activation Energy Values when $E_{A(rea)} = 30$ kcal/mol

$\theta_{1/2(rea)}$ (hr)	P^*/A_o	$E_{A(overall)}$[a] (kcal/mol)
50	0.001	10.52#
	0.005	11.26#
100	0.001	11.59
	0.005	12.48
200	0.001	12.99
	0.005	14.09
	0.01	14.72
500	0.001	14.02
	0.005	15.47
1000	0.001	14.66
	0.005	16.41
2000	0.001	15.31
	0.005	17.39•

a. Effective only for # 0-25 °C; • 5-30 °C.

TABLE 6
The Overall Activation Energy Values when $E_{A(rea)} = 40$ kcal/mol

$\theta_{1/2(rea)}$ (hr)	P^*/A_o	$E_{A(overall)}$[a] (kcal/mol)
50	0.001	14.05##
	0.005	15.59##
100	0.001	14.36#
	0.005	16.06#
200	0.001	14.67
	0.005	16.55
	0.01	17.60
500	0.001	16.56
	0.005	18.95
1000	0.001	17.58
	0.005	20.54•
2000	0.001	18.64
	0.005	22.20••

a. Effective only for # 0-25 °C; ## 0-20 °C; • 5-30 °C; •• 10-30 °C.

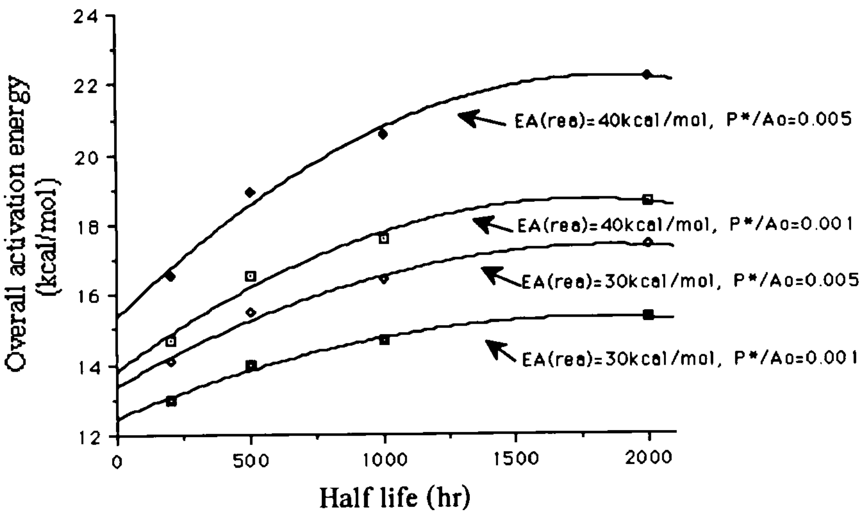

FIGURE 6
The Dependence of Overall Activation Energy for a Simultaneous Diffusion and Reaction Process on Activation Energy and Half Life of a Chemical Reaction and P*/Ao Ratio.

values depend on $E_{A(rea)}$, $\theta_{1/2(rea)}$ and P^*/A_o ratio. Generally speaking, $E_{A(overall)}$ values increase with $E_{A(rea)}$, $\theta_{1/2(rea)}$ and P^*/A_o (Fig. 6). However, for a fast reaction (at the lower end of half life), the high temperature end (30°C) will deviate from the Arrhenius model; for a slow reaction (at the higher end of half life), the lower temperature end will deviate from the model. Generally, any combined process in which $E_{A(rea)} = 40$ kcal/mol and $P^*/A_o = 0.005\text{-}0.001$, $\theta_{1/2(rea)} = 200\text{-}1000$ hr; or $E_{A(rea)} = 30$ kcal/mol and $P^*/A_o = 0.005\text{-}0.001$, $\theta_{1/2(rea)} = 1000\text{-}2000$ hr could significantly increase the $E_{A(overall)}$. The results for $E_A = 20$ kcal/mol are not shown, since the combination of the diffusion process with a chemical reaction at this activation energy did not improve the overall activation energy value within the temperature range and the rate range we were interested in.

TABLE 7
Prediction of the Combined Rate Constant for Diffusion and Reaction under Variable Time-Temperature Conditions [a]

$E_{A(rea)}$ (kcal/mol)	$\theta_{1/2(rea)}$ (hr)	$E_{A(overall)}$ (kcal/mol)	k_c(mm^2/hr) At mean temp.	k_c(mm^2/hr) Predicted from Hicks factor	k_c(mm^2/hr) Predicted from numerical integration
30	1000	14.66	1.686	1.812	2.383
	2000	15.31	1.389	1.501	1.968
40	200	14.67	2.060	2.214	3.143
	500	16.56	1.661	1.819	2.639
	1000	17.58	1.372	1.522	2.262
	2000	18.64	1.101	1.236	1.812

a. $T = T_o + a \sin(2\pi t/P_e)$ with a=6 °C, T_o=10 °C and P_e=24 hr.

Prediction under Variable Time-Temperature Conditions

The basis of a TTI is that it will be able to predict the change under variable time-temperature conditions. For example, Eq.(13) is the function for a sine wave temperature distribution.

$$T = T_o + a\mathrm{Sin}(2\pi t/P_e) \qquad (13)$$

To test the prediction, we set T_o = 10°C, amplitude a to 6°C, and period P_e to 24 hr. The solution of Eq.(12) was done by numerical intergration, using the calculated temperature T to get D and k at each time and then solving for the combined reaction and diffusion rate constant k_c. Also seen in Table 7 is the rate increase predicted based on the Hicks [14] sine wave factor and numerical integration in Eq. 12. The reliability of the actual tags using this construction needs to be tested under these conditions to see whether it will show a history effect. Appropriate chemicals to represent A and B also need to be chosen before this history effect can be determined.

Estimating the Quantity of Diffusing Material Needed

The quantity of diffusing material needed to be deposited at the origin on the tag can also be estimated from known parameters [12,15] where:

$$Q = A_O \sqrt{\frac{D}{k}} \left[\left(kt + \frac{1}{2} \right) \operatorname{erf} \sqrt{kt} + \sqrt{\frac{kt}{p}} e^{-kt} \right] \tag{14}$$

If we let

$$Y = \frac{Q}{A_O} \sqrt{\frac{k}{D}} \tag{15}$$

then Y is dimensionless. In Fig. 7, the function

$$Y = \left(kt + \frac{1}{2} \right) \operatorname{erf} \sqrt{kt} + \sqrt{\frac{kt}{p}} e^{-kt} \tag{16}$$

is plotted, which enables the quantity of diffusing materials absorbed at any time to be readily found when other factors are known. When kt is sufficiently large, Eq. 16 is reduced to:

$$Y = (kt + 1/2) \tag{17}$$

that is shown as an asymptote in Fig. 7. In other words, the rate of absorption and the concentration at any point tend to reach a constant value.

CONCLUSIONS

Theoretically, it is feasible to design a time/temperature integrator tag that has an activation energy similar to that of the food or drug it is to monitor by having the diffusing material react with another compound bound on the diffusion track. The obtained theoretical solution for a simultaneous diffusion and reaction process will give a good guide for the industry in developing such a tag. It was found that the $E_{A(\text{overall})}$ for a

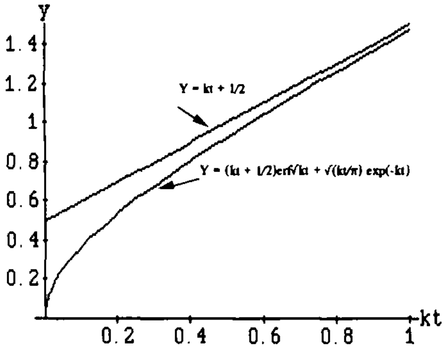

FIGURE 7
The Relationship between a Dimensionless Quantity Y (Related to the Amount of Diffusing Substance) with kt.

diffusion and reaction process depends on $E_{A(rea)}$, the reaction rate or half life of the chemical reaction and the concentration of visible product at the moving boundary. The $E_{A(overall)}$ values could be significantly changed when appropriate parameters are chosen, e.g. doubled (9.8->19 kcal/mol) when $E_{A(rea)} = 40$ kcal/mol, $\theta_{1/2(rea)} = 500$ hr and P*/Ao = 0.005. Thus, the applicability of a diffusion TTI could be enhanced. The complex equation for a simultaneous diffusion and chemical reaction process could be described numerically by $f(x) = x^2 = k_c\, t^*$ as with diffusion alone making for an easy to read tag. An equation was developed to estimate the amount of diffusing substance needed.

The remaining problems are to find some chemical compounds (A and B) that will meet the assumptions and boundary conditions and will react with each other and have the characteristics and parameters estimated previously.

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NOMENCLATURE

A	a colorless compound sitting on the end of a tag
a	amplitude in a sine wave function (°C)
A_o	initial concentration of A
B	a colorless compound bound on the wick
C*	visible concentration of diffusing material in a simple diffusion tag
C_o	initial concentration of diffusing material in a simple diffusion tag
D	diffusion coefficient (mm^2/hr)
E_A	activation energy (kcal/mol)
k	rate constant of a first order chemical reaction (hr^{-1})
k_c	combined rate constant for a simultaneous diffusion and reaction process (mm^2/hr)
k_d	rate constant for a single diffusion process (mm^2/hr)
k_o	pre-exponential factor in the Arrhenius model
P	a product of A and B with a visible color
P*	visible concentration of P
P_e	period in a sine wave function (hr)
P_o	initial concentration of P
Q	quantity of diffusing material
$\theta_{1/2}$	half life of a first order chemical reaction (hr)
θ_s	the endpoint time for either the tag or food (hr)
R	gas constant (1.987 cal/mol.K)
T	temperature (°C or K)
T_o	mean temperature in a sine wave function (°C)
t	time (hr)
t*	the time to reach P* or C* (hr)
x	the distance of diffusion on a wick (mm)
Y	a dimensionless quantity

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